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One-way cloak based on nonreciprocal photonic crystal

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We propose a physical concept of non-reciprocal transformation optics, by which a one-way invisible cloak is designed. The one-way invisible cloak is made of a coordinate-transformed nonreciprocal photonic crystal, showing a perfect cloaking for wave incident from one direction but acting as a perfect reflector for wave from the counter direction. The proposed design shows a high promise of applications in military, as protecting the own information to be detected but efficiently grabbing the information from the "enemy" side. © *2011 American Institute of Physics*. [doi:10.1063/1.3648112]

Transformation optics has recently drawn much attention due to its ability of controlling the pathway and filed distribution of electromagnetic (EM) waves.[1–4] The propagation of waves can be manipulated by the transformation of the space coordinate and the space dependent permittivity and permeability, which induces abundant novel effects from cloaking, illusion optics, to artificial black holes for different kinds of waves.[5–13] Among them, the most promising one is the invisible cloak, where light is bent around the concealed region to make inside object "invisible." However, so far properties drawn by the transformation method are performed on homogeneous medium, and the reciprocity of light makes the cloak invisible to both sides, strongly limiting its practical use in military since even the side launching the cloak cannot detect where the cloak goes. Therefore, it is highly desirable to introduce the breaking of reciprocity and create a one-way cloak only for the "enemy" side.

In this letter, we utilize a two-dimensional (2D) nonreciprocal photonic crystal (NRPC)[14–19] inherently associated with the break of the parity (P) and time-reversal (T) symmetries; a one-way cloak is designed and exhibits distinguished cloaking characteristics for EM waves incident from two different sides: one is completely cloaked in NRPC and the other is totally reflected. The directions of cloaking and reflecting could also be reversed.

Herein, we introduce one kind of square-shaped coordinate transformation method based on the NRPC. Fig. 1(a) shows the schematic of a 2D 4×4 NRPC made of a square lattice of yttrium-iron-garnet (YIG) semi-cylinders embedded in air. The YIG semi-cylinders have permittivity of $\varepsilon = 15\varepsilon_0$ and the radius of $r = 0.3a$, where a is the lattice constant. An external 1600 Gauss dc magnetic field (along $+z$ direction) is applied to induce the permeability into a gyromagnetic form

$$\vec{\mu} = \begin{bmatrix} \mu & i\kappa & 0 \\ -i\kappa & \mu & 0 \\ 0 & 0 & \mu_0 \end{bmatrix} = \vec{\mu}_d - i\kappa\hat{z} \times \vec{I}, \tag{1}$$

where $\mu = 14\mu_0$, $\kappa = 12.4\mu_0$ at 4.28 GHz,[20] and $\vec{\mu}_d$ is the diagonal part. It is necessary for the nonreciprocal propagation that P and T symmetries should be simultaneously broken. Herein, the introduced gyro-magnetic permeability tensor breaks the T symmetry, while the asymmetric semi-cylindrical shape intrinsically induces the break of P symmetry in the x direction. The break of both P and T symmetries induce the nonreciprocal propagation of EM waves, showing a distinguished one-way feature for the counter-propagating bulk waves with $\omega(k) \neq \omega(-k)$, where k and $-k$ represent the counter-propagating wave vectors. A square-shaped coordinate transformation is thus introduced into modulate NRPC to design a one-way cloak with EM waves only propagating within the compressed space between S_1 and S_2. The square-shaped transformation equations to map the region $\{\max(|x|,|y|) \le S_1\}$ onto the region $\{\max(|x|,|y|) \le S_1$ and $\min(|x|,|y|) \ge S_2\}$, for shadowed region in Fig. 1(a), can be written as $\{x' = Ax + S_2, y' = yx'/x, z' = z\}$, where $A = (S_1 - S_2)/S_1$, $S_1 = 2a$ and $S_2 = a$. After transformation, the semi-cylinders change to some anomalous shapes in the new space [Fig. 1(b)]. The permittivity and permeability yield

$$\begin{aligned} \vec{\varepsilon}(x', y') &= B_{3\times3}\vec{\varepsilon}(x, y) \\ \vec{\mu}(x', y') &= B_{3\times3}\vec{\mu}_d(x, y) - i\kappa(x, y)\hat{z} \times \vec{I}, \end{aligned} \tag{2}$$

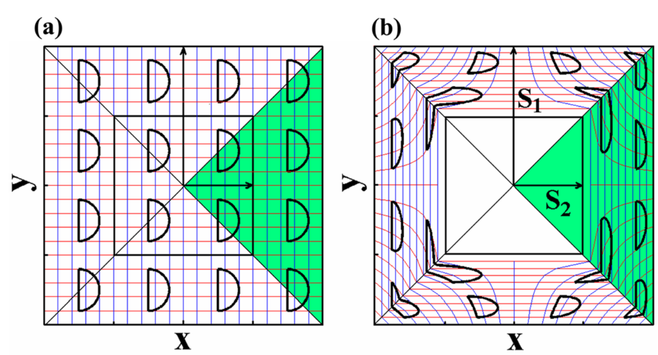

FIG. 1. (Color online) Spatial coordinate transformation for the square-shaped cloak based on the nonreciprocal gyrotropic photonic crystal (a) original space and (b) transformed space.

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Where

$$B_{3\times 3} = \begin{bmatrix} 1 - S_2/x' & -S_2y'/x'^2 & 0 \\ -S_2y'/x'^2 & [1 + (S_2y'/x'^2)^2]/(1 - S_2/x') & 0 \\ 0 & 0 & (1 - S_2/x')S_1^2/(S_1 - S_2)^2 \end{bmatrix}.$$

In Eq. (2), both permittivity and permeability are still periodic functions of the position in the original space. It should be noticed that the coordinate transformation in the xy plane does not change the distribution of imaginary part of permeability. All the following numerical results are calculated by using a finite element method software package (COMSOL MULTIPHYSICS 3.3).

The coordinate transformation does not change the original physical properties of the original space. Therefore, the band diagram of TM mode (electric field out of xy plane) in the original NRPC shown in Fig. 2(a) also represents the band structure of the transformed NRPC cloak, where the solid and dot lines represent the opposite high-symmetry directions, respectively, shown in the inset. The numerical simulation of the waves propagation at two different frequencies 0.31 $(2\pi c/a)$(in the pass band [Fig. 2(b)]) and 0.22 $(2\pi c/a)$(in the band gap) [Fig. 2(c)] is consistent with the calculated band structure: a perfect cloaking is mapped in pass band while total reflection in the band gap.

In Fig. 2(a), the dispersion relations along two opposite high-symmetry directions in the fifth and the sixth bands are different, clearly showing nonreciprocal characters. The equi-frequency surfaces (EFSs) in the sixth band are plotted in Fig. 3(a), where the white and black circles represent the EFSs at the frequency of 0.51 $(2\pi c/a)$ in NRPC and in air, respectively. A pair of counter incidences are indicated by $+30°$ and $-30°$ arrows, respectively. Group velocity and phase velocity are indicated by V_g and V_p, respectively. The k incidence from $+30°$ has a projection intersection with the EFS of NRPC, suggesting propagation through NRPC, whereas the counter incident wave from k' along $-30°$ is unable to meet this momentum-match condition, indicating a total reflection. A pair of counter incident EM waves has completely different propagation features through the NRPC cloak: one is perfectly cloaked by the NRPC [Fig. 3(b)] but the other is totally reflected [Fig. 3(c)]. Because the k-space symmetry is preserved relative to the y axis, propagation of waves is expected to be symmetric as well corresponding to the y axis as validated in Figs. 3(d) and 3(e). This semi-cylindrical NRPC has the even real part of permeability but the odd imaginary part along the y direction along the y direction, suggesting PT symmetry along the y direction [Figs. 3(b) vs. 3(d) and Figs. 3(c) vs. 3(e)].[19,21]

In the fifth band, the NRPC and its based cloak exhibits completely reversed one-way characters, compared to those in the sixth band. The EFSs in the fifth band are shown in Fig. 4(a), where the white and black circles correspond to the EFS at the frequency of 0.48 $(2\pi c/a)$ in NRPC and in air, respectively. As seen from Fig. 4(a), transmission is only allowed for the incident wave vector k' from $-30°$ since the projection of k' can intersect with the EFS in NRPC. Consistent with the EFSs, the propagation of EM waves in Figs. 4(b) and 4(c) demonstrate the expected one-way cloaking, while Figs. 4(d) and 4(e) show the corresponding PT symmetric case with reverse one-way characters.

Additional coordinate transformation is implemented to make the one-way cloaking also available for normal incidences, which typically has the most practical impacts regarding the application perspectives. Since nonreciprocal characters of NRPC are mostly valid for oblique incidences, we rotate our NRPC cloak 45° to have the local interface of the cloak 45° oblique. A parallelogram-shaped transformation is used based on a 4×5 NRPC and the right region follows the equations: $x' = Ax + S_2x/[x - y\tan(\pi/4)]$; $y' = Ay + S_2y/[x - y\tan(\pi/4)]$; $z' = z$.

Two frequencies of 0.51 $(2\pi c/a)$ and 0.48$(2\pi c/a)$ display the different one-way cloaking behaviors, as shown in Figs. 5(a) and 5(b), respectively. The direction of the cloaking can be effectively controlled by adjusting the operation frequency, which provides much freedom in systematic manipulation of wave propagation.

In conclusion, we have proposed and designed a one-way invisible cloak for EM waves based on a 2D NRPC with simultaneously broken P and T symmetries. Numerical simulations have been applied to validate the existence of the one-way cloaking at different frequencies, which agrees well with the nonreciprocal band structures of NRPC. Two different types of coordinate transformation have been utilized to achieve the cloaking for both normal and oblique incidences.

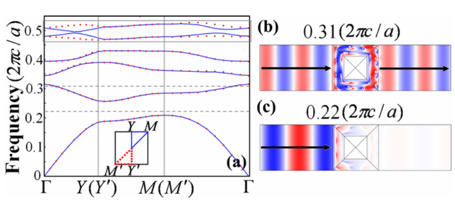

FIG. 2. (Color online) (a) Band structure of TM mode of the designed semi-cylindrical YIG PC. The solid and dot lines represent two sets of opposite high-symmetry directions, respectively, as shown in the inset. (b) Electric field distribution at the frequency of 0.31$(2\pi c/a)$ in pass band. (c) Electric field distribution at frequency of 0.22$(2\pi c/a)$ in band gap.

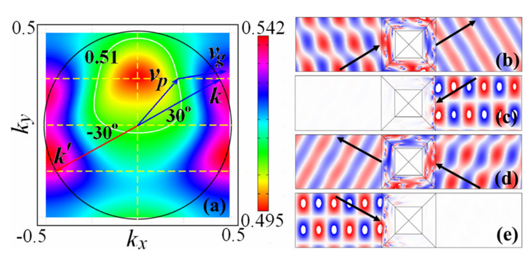

FIG. 3. (Color online) (a) EFSs in the sixth band of NRPC, where the white and black circles correspond to the frequency of 0.51$(2\pi c/a)$ in NRPC and in air, respectively. A pair of counter incidences are indicated by $+30°$ and $-30°$ arrows, respectively. Group velocity and phase velocity are indicated by V_g and V_p, respectively. The corresponding electric filed distributions of the counter incident EM waves are shown in (b) $+30°$ and (c) $-30°$. (d) and (e) show the PT symmetric cases of (b) and (c), respectively.

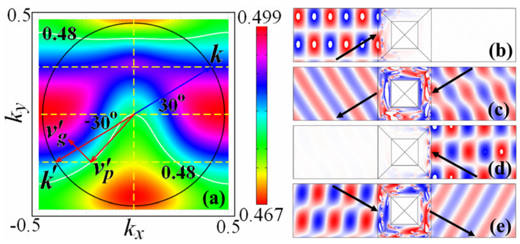

FIG. 4. (Color online) (a) EFSs in the fifth band at the frequency of $0.48(2\pi c/a)$. The corresponding electric filed distributions are shown in (b) $+30°$ and (c) $-30°$. (d) and (e) show the *PT* symmetric cases of (b) and (c), respectively.

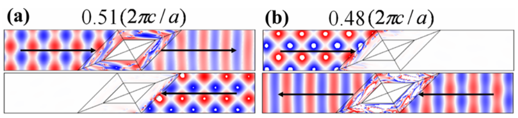

FIG. 5. (Color online) Parallelogram-shape transformed cloak at two different frequencies: (a) $0.51(2\pi c/a)$ and (b) $0.48(2\pi c/a)$.

This one-way cloaking effect can be more systematically controlled (on, off, or completely reversed) by adjusting the operation frequencies or changing the direction of the external magnetic field.

From the point of experimental view, this proposed structure can be fabricated conveniently by drilling inhomogeneous holes to average the local effective indices.[9,10] Therefore, due to uniformity of off-diagonal elements of the permeability tensor, the applied magnetic field can be easily set a constant, which can make experimental implementation much more straightforward. The method by introducing the transformation optics into the non-uniform space might find a way to explore some one-way compact applications in solid-state photo-electronic devices and can definitely induce a number of interesting physical phenomena and applications. For example, it is expected to realize a one-way optical black hole that only supports inward propagation without any backward response, construct a one-way superlens that only provides focusing with sub-diffraction limited information to one side, and so on.

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[1]J. B. Pendry, D. Schurig, and D. R. Smith, Science **312**, 1780 (2006).
[2]U. Leonhardt, Science **312**, 1777 (2006).
[3]U. Leonhardt, Science **323**, 110 (2009).
[4]H. Chen, C. T. Chan, and P. Sheng, Nature Mater. **9**, 387 (2010).
[5]M. Rahm, D. Schurig, D. A. Roberts, S. A. Cummer, D. R. Smith, and J. B. Pendry, Photonics Nanostruct. Fundam. Appl. **6**, 87 (2008).
[6]J. Li and J. B. Pendry, Phys. Rev. Lett. **101**, 203901 (2008).
[7]Y. Lai, H. Chen, Z. Q. Zhang, and C. T. Chan, Phys. Rev. Lett. **102**, 093901 (2009).
[8]J. Hao, W. Yan, and M. Qiu, Appl. Phys. Lett. **96**, 101109 (2010).
[9]J. Valentine, J. Li, T. Zentgraf, G. Bartal, and X. Zhang, Nature Mater. **8**, 568 (2009).
[10]L. H. Gabrielli, J. Cardenas, C. B. Poitras, and M. Lipson, Nature Photon. **3**, 461 (2009).
[11]R. Liu, C. Ji, J. J. Mock, J. Y. Chin, T. J. Cui, and D. R. Smith, Science **323**, 366 (2009).
[12]Q. Cheng, T. J. Cui, W. X. Jiang, and B. G. Cai, New J. Phys. **12**, 063006 (2010).
[13]Y. Lai, J. Ng, H. Y. Chen, D. Z. Han, J. J. Xiao, Z. Q. Zhang, and C. T. Chan, Phys. Rev. Lett. **102**, 253902 (2009).
[14]Z. Yu, Z. Wang, and S. Fan, Appl. Phys. Lett. **90**, 121133 (2007).
[15]M. Vanwolleghem, X. Checoury, W. Smigaj, B. Gralak, L. Magdenko, K. Postava, B. Dagens, P. Beauvillain, and J. M. Lourtioz, Phys. Rev. B **80**, 121102(R) (2009).
[16]F. D. M. Haldane and S. Raghu, Phys. Rev. Lett. **100**, 013904 (2008).
[17]Z. Wang, Y. D. Chong, J. D. Joannopoulos, and M. Soljačić, Phys. Rev. Lett. **100**, 013905 (2008).
[18]Z. Wang, Y. D. Chong, J. D. Joannopoulos, and M. Soljačić, Nature (London) **461**, 772 (2009).
[19]C. He, M. H. Lu, X. Heng, L. Feng, and Y. F. Chen, Phys. Rev. B **83**, 075117 (2011).
[20]D. M. Pozar, *Microwave Engineering*, 2nd ed. (Wiley, New York, 1998).
[21]K. G. Makris, R. El-Ganainy, D. N. Christodoulides, and Z. H. Musslimani, Phys. Rev. Lett. **100**, 103904 (2008).